MAXIMUM-NORMAL-STRESS THEORY FOR BRITTLE MATERIALS

THE MAXIMUM-NORMAL-STRESS (MNS) THEORY STATES THAT FAILURE OCCURS WHENEVER ONE OF THE THREE PRINCIPAL STRESSES EQUALS OR EXCEEDS THE STRENGTH.

THE FAILURE CRITERION FOR THE MNS THEORY IS:

$$n = \frac{S_{ut}}{\sigma_1} \quad \text{OR} \quad n = \frac{-S_{uc}}{\sigma_3}$$

where S_{ut} & S_{uc} ARE THE ULTIMATE TENSILE & COMPRESSIVE STRENGTHS, RESPECTIVELY.

and $\sigma_1 \geq \sigma_2 \geq \sigma_3$ ARE THE ORDERED PRINCIPAL STRESSES

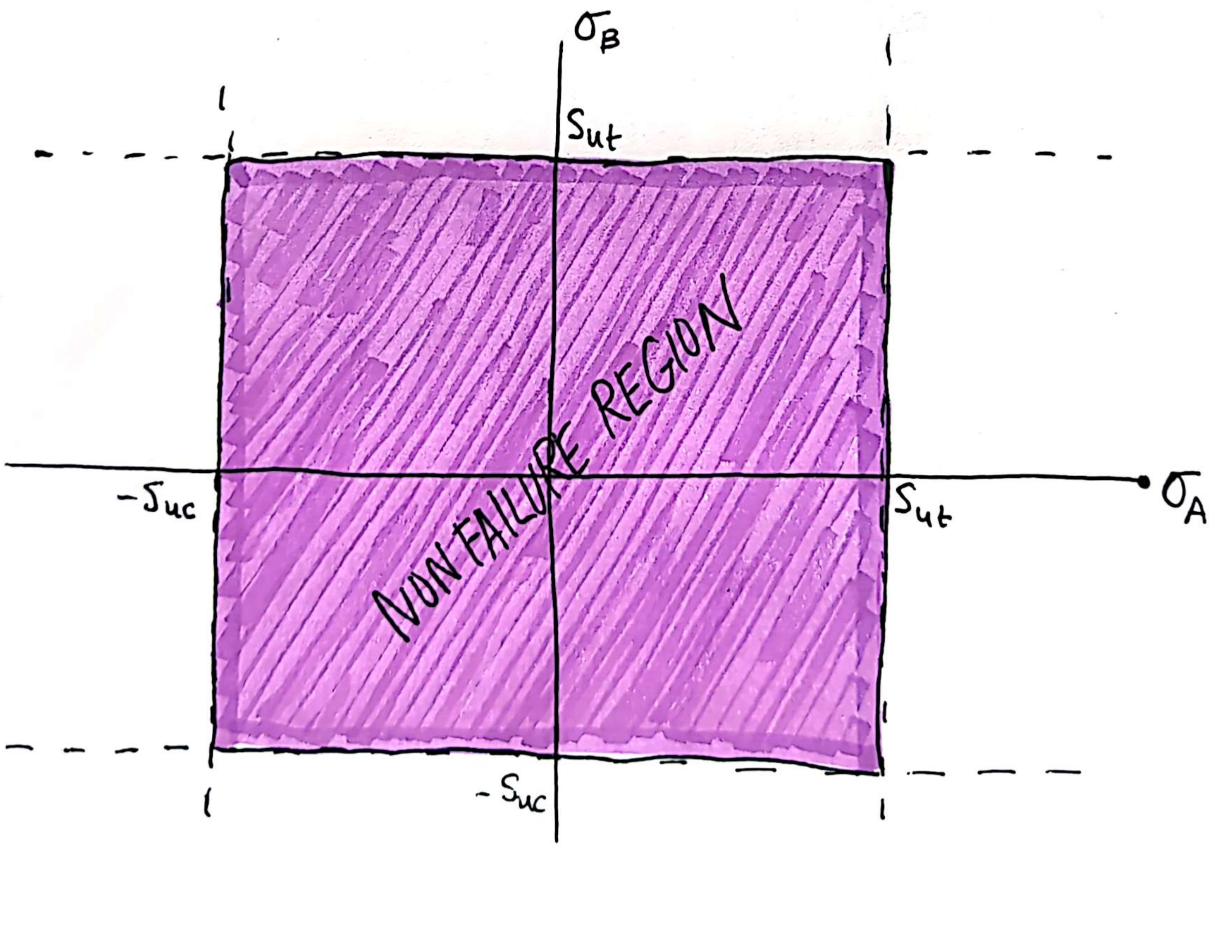
FOR A PLANE STRESS STATE, ONE OF THE PRINCIPAL STRESSES IS ZERO. THERE ARE 2 POSSIBILITIES:

1. $|\sigma_A| \geq |\sigma_B|$ $(\sigma_1 \geq \sigma_2 \geq 0$ OR $\sigma_1 \geq 0 \geq \sigma_3)$

   THEN, THE MNS FAILURE CRITERION IS $\sigma_A \geq S_{ut}$ $\left(n = \frac{S_{ut}}{\sigma_A} = \frac{S_{ut}}{\sigma_1}\right)$

2. $|\sigma_B| \geq |\sigma_A|$ $(0 \geq \sigma_2 \geq \sigma_3$ OR $\sigma_1 \geq 0 \geq \sigma_3)$

   THEN, THE MNS FAILURE CRITERION IS $\sigma_B \leq -S_{uc}$ $\left(n = \frac{-S_{uc}}{\sigma_B} = \frac{-S_{uc}}{\sigma_3}\right)$



MODIFICATIONS OF THE MOHR THEORY FOR BRITTLE MATERIALS

WE WILL DISCUSS 2 MODIFICATIONS OF THE MOHR THEORY FOR BRITTLE MATERIALS: THE BRITTLE-COULOMB-MOHR THEORY AND THE MODIFIED MOHR THEORY.

THE FAILURE CRITERION FOR THE BRITTLE-COULOMB-MOHR (BCM) THEORY IS AS FOLLOWS FOR PLANE STRESS:

$\left(n = \frac{S_{ut}}{\sigma_A}\right)$ $\boxed{\sigma_A \geq S_{ut}}$ WHEN $\sigma_A \geq \sigma_B \geq 0$ $(\sigma_1 \geq \sigma_2 \geq 0)$

$\left(\frac{1}{n} = \frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}}\right)$ $\boxed{\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} \geq 1}$ WHEN $\sigma_A \geq 0 \geq \sigma_B$ $(\sigma_1 \geq 0 \geq \sigma_3)$

$\left(n = \frac{-S_{uc}}{\sigma_B}\right)$ $\boxed{\sigma_B \leq -S_{uc}}$ WHEN $0 \geq \sigma_A \geq \sigma_B$ $(0 \geq \sigma_2 \geq \sigma_3)$

THE FAILURE CRITERION FOR THE MODIFIED MOHR (MM) THEORY IS AS FOLLOWS FOR PLANE STRESS:

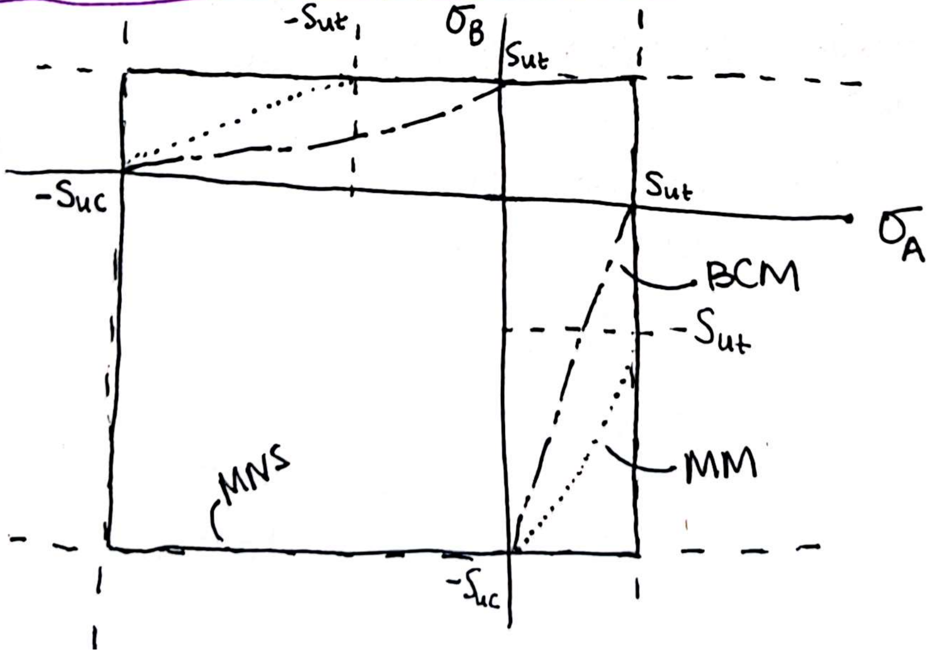
$\left(n = \frac{S_{ut}}{\sigma_A}\right)$ $\sigma_A \geq S_{ut}$ WHEN $\sigma_A \geq \sigma_B \geq 0$

$\left(n = \frac{-S_{uc}}{\sigma_B}\right)$ $\sigma_B \leq -S_{uc}$ WHEN $0 \geq \sigma_A \geq \sigma_B$

} SAME AS BCM

$\left(n = \frac{S_{ut}}{\sigma_A}\right)$ $\sigma_A \geq S_{ut}$ WHEN $\sigma_A \geq 0 \geq \sigma_B$ AND $|\sigma_A| \geq |\sigma_B|$

$\left(\frac{1}{n} = \frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc} S_{ut}} - \frac{\sigma_B}{S_{uc}}\right)$ $\frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc} S_{ut}} - \frac{\sigma_B}{S_{uc}} \geq 1$ WHEN $\sigma_A \geq 0 \geq \sigma_B$ AND $|\sigma_B| \geq |\sigma_A|$

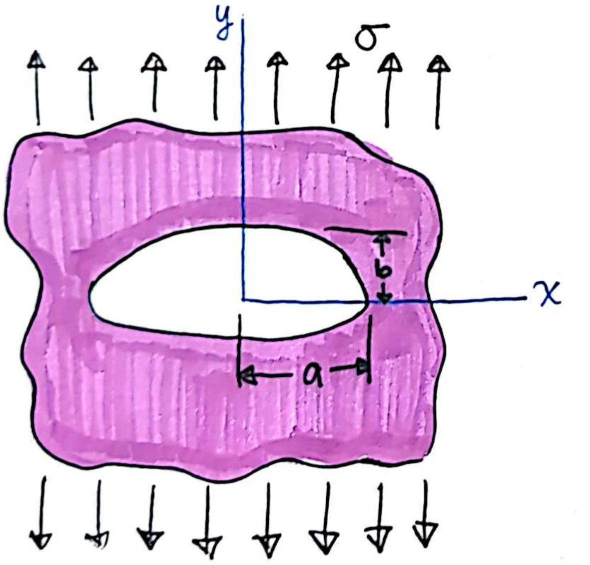


INTRODUCTION TO FRACTURE MECHANICS

SUDDEN BRITTLE FRACTURE OF SO-CALLED DUCTILE MATERIALS IS POSSIBLE, AND DESIGNERS MUST BE AWARE OF THIS DANGER.

LINEAR-ELASTIC FRACTURE MECHANICS (LEFM) IS A THEORY USED TO PREDICT THE BEHAVIOR OF CRACKS IN MATERIALS UNDER STRESS.

FOR AN INFINITE PLATE LOADED BY AN APPLIED UNIAXIAL STRESS σ, THE MAXIMUM STRESS OCCURS AT $(\pm a, 0)$ AND IS GIVEN BY



$$(\sigma_y)_{max} = \left(1 + 2\frac{a}{b}\right)\sigma$$

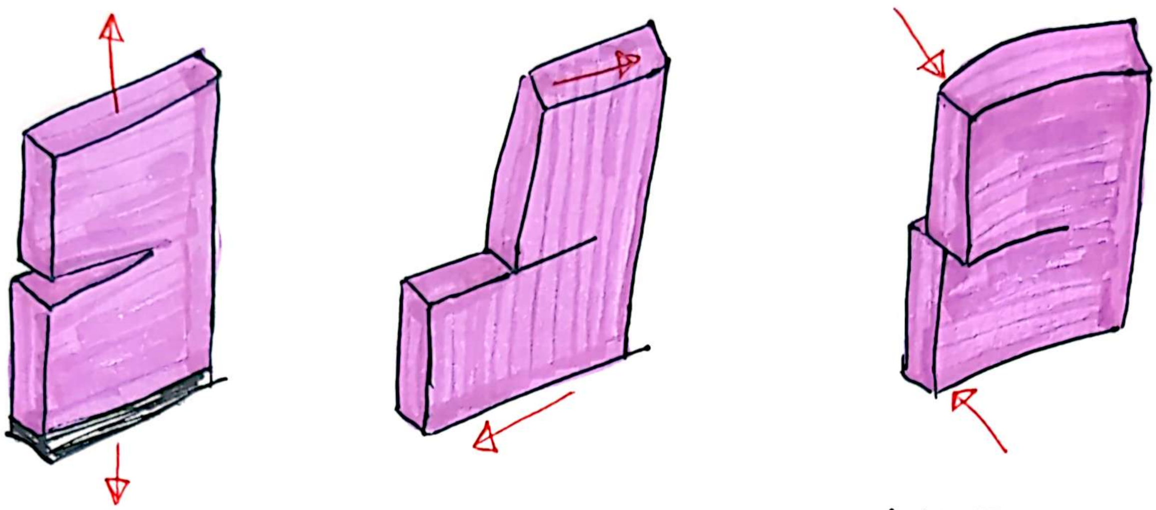
(NOTE FOR A CIRCULAR CRACK, A=B AND
$$(\sigma_y)_{max} = 3\sigma$$
)

FOR A MICROSCOPICALLY FINE CRACK, $b \to 0$ AND $(\sigma_y)_{max} \to \infty$. THIS INFINITELY SHARP CRACK IS PHYSICALLY IMPOSSIBLE, AND THIS OBSERVATION INSPIRED GRIFFITH'S WORK ON CRACK GROWTH (~1920). IRWIN EXPANDED GRIFFITH'S WORK BY INTRODUCING THE STRESS INTENSITY FACTOR, WHICH IS GIVEN BY

$$K_I = \sigma\sqrt{\pi a} \qquad [\text{UNITS ARE } MPa\sqrt{m} \text{ OR } ksi\sqrt{in}]$$

FOR A $\underbrace{\text{MODE I CRACK}}_{\text{(INDICATED BY THE SUBSCRIPT)}}$ IN AN INFINITE PLATE.

THERE ARE 3 DISTINCT CRACK PROPAGATION MODES



MODE I: OPENING   MODE II: SLIDING   MODE III: TEARING

MODE I IS THE MOST COMMON, SO WE WILL FOCUS ON THIS MODE FOR OUR CLASS.

THE STRESS INTENSITY FACTOR IS A FUNCTION OF GEOMETRY, SIZE & SHAPE OF THE CRACK, AND THE TYPE OF LOADING. FOR LOAD AND GEOMETRIC CONFIGURATIONS,

$$\boxed{K_I = \beta \sigma \sqrt{\pi a}}$$

WHERE β = STRESS INTENSITY MODIFICATION FACTOR

WHEN K_I REACHES A CRITICAL VALUE (K_{Ic}), CRACK PROPAGATION INITIATES. THE CRITICAL STRESS INTENSITY FACTOR, K_{Ic}, IS A MATERIAL PROPERTY (ALSO CALLED THE FRACTURE TOUGHNESS OF THE MATERIAL) THAT DEPENDS ON THE MATERIAL, CRACK MODE, PROCESSING OF THE MATERIAL, TEMPERATURE, LOADING RATE, AND THE STATE OF STRESS AT THE CRACK SITE.

THE STRENGTH-TO-STRESS RATIO CAN BE USED AS A FACTOR OF SAFETY

$$\boxed{n = \frac{K_{Ic}}{K_I}}$$